

## Convection Heat Transfer

Heat transfer from a solid to the surrounding fluid

Consider fluid motion

Recall flow of water in a pipe

Thermal Boundary Layer

- A temperature profile similar to velocity profile. Temperature of pipe surface is kept constant. At the end of the thermal entry region, the boundary layer extends to the center of the pipe.

Therefore, two boundary layers: hydrodynamic boundary layer and a thermal boundary layer.

Analytical treatment is beyond the scope of this course. Instead we will use an empirical approach.

Drawback of empirical approach: need to collect large amount of data.

**Reynolds Number:**

**Nusselt Number:** it is the dimensionless form of convective heat transfer coefficient.

Consider a layer of fluid as shown

If the fluid is stationary, then

And

Dividing

Replacing  $l$  with a more general term for dimension, called the characteristic dimension,  $d_c$ , we get

$$N_{Nu} \equiv \frac{hd_c}{k}$$

Nusselt number is the enhancement in the rate of heat transfer caused by convection over the conduction mode. If  $N_{Nu} = 1$ , then there is no improvement of heat transfer by convection over conduction. On the other hand, if  $N_{Nu} = 10$ , then rate of convective heat transfer is 10 times the rate of heat transfer if the fluid was stagnant.

**Prandtl Number:** It describes the thickness of the hydrodynamic boundary layer compared with the thermal boundary layer. It is the ratio between the molecular diffusivity of momentum to the molecular diffusivity of heat.

$$N_{Pr} = \frac{\text{kinematic viscosity}}{\text{thermal diffusivity}} = \frac{\nu}{\alpha}$$

$$N_{Pr} = \frac{\mu C_p}{k}$$

If  $N_{Pr} = 1$  then the thickness of the hydrodynamic and thermal boundary layers will be the same. For gases  $N_{Pr} = 0.7$  and for water it is equal to 10.

**Experimental Approach:**

Consider fluid flowing in a heated pipe. We are interested in convective heat transfer from the inside heated surface of the pipe to the fluid. Carry out the experiment with fluid entering at velocity of  $u_1$  at a temperature  $T_i$ . Measure the electric current passing through to determine the rate of heat transfer,  $q$ . Thus, we determine  $q$ ,  $A$ ,  $T_i$ , and  $T_s$ . Then using Newton's law of cooling we calculate  $h$ .

Now repeat the experiment for different diameter of pipe, or different temperature of the pipe surface, and determine new h value. Thus perform a series of experiments to determine h values as a function of operating variables, q, A, T<sub>i</sub> and T<sub>s</sub>.

This can result in a large amount of data, but we organize data using the three dimensionless numbers.

For each experimental set, we calculate the dimensionless numbers and using a log-log plot, we plot Nusselt Number vs Reynolds number for different values of Prandtl number. It has been experimentally shown that for a given fluid, with a fixed Prandtl number, we get a straight line plot between log N<sub>Nu</sub> and log N<sub>Re</sub>. Thus a convenient expression is of the form.

By substituting experimentally obtained coefficients in above equation, we obtain empirical correlations.

Different correlations have been obtained for laminar and turbulent conditions

### **Methodology to solve problems involving convection heat transfer:**

A suggested methodology to solve problems requiring calculations of convective heat transfer coefficient using empirical correlations is as follows:

#### **1. Identify flow geometry**

Is it a pipe, sphere, rectangular duct, plate? Is the fluid flowing inside or outside the pipe

#### **2. Identify the fluid, and determine its properties**

Is it water, air or a liquid food? Determine the average fluid temperature far away from the solid surface, T<sub>∞</sub>

. In some cases, the average inlet and outlet temperatures may be different, in that case calculate the average fluid temperature.

$$T_{\infty} = \frac{T_i + T_e}{2}$$

Use the average fluid temperature to determine the physical and thermal properties of the fluid, such as viscosity, density, and thermal conductivity (from Appendix tables).

#### **3. Calculate the Reynolds number**

Using the velocity of the fluid, fluid properties, characteristic dimension, determine the Reynolds number.

Determine if the flow is laminar, transitional, or turbulent.

#### **4. Select an appropriate empirical correlation and determine h.**

Select an empirical correlation from the following. Calculate Nusselt number and then h value.

### **Empirical Correlations**

#### **Forced Convection**

##### **Laminar Flow in Pipes**

1. Fully developed condition with constant surface temperature

$$N_{Nu} = 3.66$$

2. Fully developed condition with uniform heat flux

$$N_{Nu} = 4.36$$

3. For both entry region and fully developed flow conditions

$$N_{Nu} = 1.86 \left( N_{Re} \times N_{Pr} \times \frac{D_c}{L} \right)^{0.33} \left( \frac{\mu_b}{\mu_w} \right)^{0.14}$$

##### **Transition flow in Pipes**

$$N_{Nu} = \frac{(f/8)(N_{Re} - 1000)N_{Pr}}{1 + 12.7(f/8)^{1/2}(N_{Pr}^{2/3} - 1)}$$

where

$$f = \frac{1}{(0.790 \ln N_{Re} - 1.64)^2}$$

#### **Turbulent Flow in Pipes**

$$N_{Nu} = 0.023 N_{Re}^{0.8} \times N_{Pr}^{0.33} \times \left( \frac{\mu_b}{\mu_w} \right)^{0.14}$$

#### **Convection in Noncircular ducts**

Calculate an equivalent diameter for use as characteristic dimension

$$D_e = \frac{4 \times \text{free area}}{\text{wetted perimeter}}$$

#### **Flow past Immersed Objects**

For example, flow past a single sphere

$$N_{Nu} = 2 + 0.60 N_{Re}^{0.5} \times N_{Pr}^{1/3} \quad \text{for} \quad \begin{cases} 1 < N_{Re} < 70,000 \\ 0.6 < N_{Pr} < 400 \end{cases}$$

Fluid properties are evaluated at

$$T_f = \frac{T_s + T_\infty}{2}$$

#### **Free Convection**

Fluid flow due to natural buoyancy of fluid. The empirical expression is of the form

$$N_{Nu} = \frac{hD_c}{k} = a(N_{Ra})^m$$

where  $a$  and  $m$  are constants;  $N_{Ra}$  is the Rayleigh number. Rayleigh number is a product of two dimensionless numbers, Grashof number and Prandtl number.

$$N_{Ra} = N_{Gr} \times N_{Pr}$$

The Grashof Number is defined as

$$N_{Gr} = \frac{D_c^3 \rho^2 g \beta \Delta T}{\mu^2}$$

where  $d_c$  is characteristic dimension (m);  $\rho$  is density ( $\text{kg/m}^3$ );  $g$  is acceleration due to gravity ( $9.80665 \text{ m/s}^2$ );  $\beta$  is coefficient of volumetric expansion ( $\text{K}^{-1}$ );  $\Delta T$  is temperature difference between wall and the surrounding bulk ( $^\circ\text{C}$ ); and  $\mu$  is viscosity (Pa s).

Grashof number is a ratio between the buoyancy forces and viscous forces. The fluid properties are determined at the average film temperature.